Reply:

Thank you for the opportunity to reply to the letter of Tien and Ramarao regarding our recent publication in your journal.[1] In general, their comments detail issues of semantics rather than any fundamental criticisms of the underlying theoretical approach. Their response reflects the long running opposition held by some in the solid-liquid separation community to the phenomenological approach to modeling compressible materials.[2, 3] The aim of our work was not to invalidate or disparage the volume-averaged Darcian approach, but rather to investigate the material properties and operating conditions required for nontraditional behavior. It has been shown in other articles[4-7] that the parameters used by the local and volume-averaged approaches are related, such that the conclusions made in Stickland et al., 2005[1], also apply to the volume-averaged approach. It is our intention to bridge the divide in this area rather than deepen it, and our article, although not attempting to discuss this issue, further highlights the inadequacy of both approaches for some industrial realities.

Tien and Ramarao's remarks concern the assumptions made in the mathematical model, specifically that the initial solids concentration, ϕ_O, is above the gel point, ϕ_g, and that the membrane resistance R_m, is negligible. They are correct to state that the process as we have modeled it follows the definition of consolidation by Shirato et al., 1970[8], such that $\phi_O > \phi_g$. However, consistent with the application of the term industrially, we consider the overall process (regardless of ϕ_O) as filtration, rather than the ambiguous term "expression". The transient behavior of the filtrate volume $V(t)$, during constant pressure filtration is divided into cake formation, represented by linear t vs. V^2 behavior, and cake compression, represented by deviation from this behavior. The approximation of Landman and White, 1997,[9] shows that linear t vs. V^2 behavior is expected even for $\phi_O > \phi_g$ - an attribute that forms the basis for most filtration analysis techniques to measure permeability.[10,11] So, while the value of ϕ_0 relative to ϕ_g is important from the perspective of mathematical formulation, from an experimental point of view, an understanding of the transition from linear dominated t versus V^2 to nonlinear cake compression behavior is more relevant, especially as few industrial filtration practitioners even measure ϕ_g.

The reasons for linear t vs. V^2 behavior for ϕ_O both above and below ϕ_g are illustrated in two filter cake volume fraction distribution plots presented here. In the $\phi_O < \phi_g$ case, the top of the cake is at ϕ_g, and the exact solution is linear t vs. V^2 up until the cake reaches the piston on its way down (see Figure 1).

In the $\phi_O > \phi_g$ case, all the material is initially networked. However, approximately linear t vs. V^2 behavior arises due to the presence of steeply changing $d\phi/dz$ (see Figure 2). When the membrane resistance is negligible compared to the cake resistance, the boundary and initial conditions, $\phi(0,t) = \phi_\infty$ and $\phi(z,0) = \phi_O$, are mutually exclusive and cake growth represents the propagation of the shock that initially forms at the membrane. Material above this consolidating material remains at ϕ_O until the cake reaches the piston. When the membrane resistance is included, the top of the cake is at the piston since $d\phi/dz \to 0$.

In our article we restricted our analysis to the $\phi_O > \phi_g$ case because the finite difference method requires special care at the discontinuity ($\phi(z_c^-,t) = \phi_g$ and $\phi(z_c^+,t) = \phi_O$) at the top of the cake for the $\phi_O < \phi_g$ case. The exact solution for the $\phi_O < \phi_g$ case is given by the similarity solution of Landman and White, 1997[9]. The conclusion that relatively long compression times and short formation times are expected when $D(\phi)$ is significantly decreasing over the range $\phi_g < \phi < \phi_\infty$ still holds.

Tien and Ramarao are correct to state that t is not initially quadratic with V. t is initially quadratic with V only when the membrane resistance is negligible, the pressure is applied instantaneously, and the sedimentation timescale is much greater than the filtration timescale. However, these effects are generally short-lived and are insignificant to the work in question, which investigated long-time effects. Models that include these effects are presented elsewhere.[12-14] We query Tien and Ramarao's statement that the work of Atsumi and Akiyama[15] is the only rational approach to analyzing cake filtration, since they do not consider these effects either.

Finally, we disagree with Tien and Ramarao that it is not surprising that nontraditional behavior was observed. As stated earlier, linear t vs. V^2 behavior is expected for consolidation. By defining cake formation and compression in terms of the transient behavior, our article outlines the necessary material properties and initial conditions for the behavior to show negligible cake formation times relative to the cake compression times, which is still a novel result for consolidation.

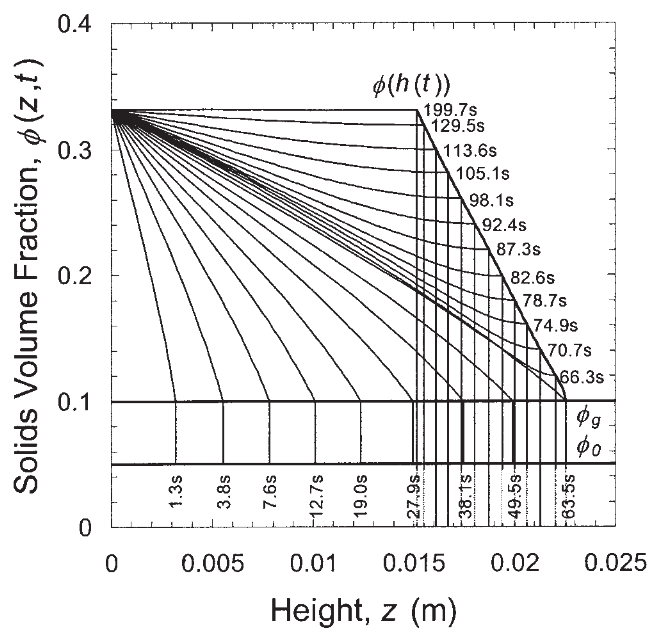

Figure 1. Example of solids distribution plot for initial suspension un-networked ($P_y(\phi) = 10((\phi/\phi_g)^5 - 1)$ Pa, $R(\phi) = 10^9(1-\phi)^{-3.5}$ Pa.s/m², ϕ_g = 0.1 v/v, ϕ_O = 0.05 v/v, h_O = 0.1 m, ΔP = 4 kPa).

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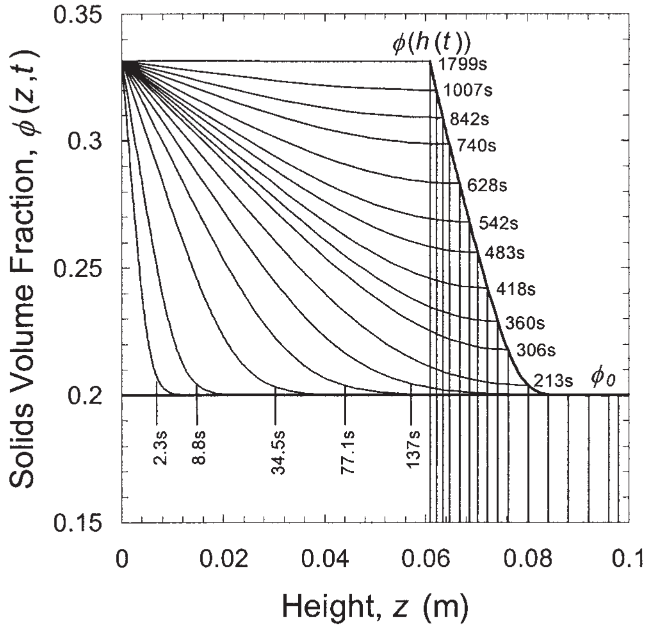

Figure 2. Example of solids distribution plot for initial suspension networked ($P_y(\phi) = 10((\phi/\phi_g)^5 - 1)$ Pa, $R(\phi) = 10^9(1-\phi)^{-3.5}$ Pa.s/m^2, $\phi_g = 0.1$ v/v, $\phi_o = 0.2$ v/v, $h_o = 0.1$ m, $\Delta P = 4$ kPa)

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